Introduction to inference in pulsar timing

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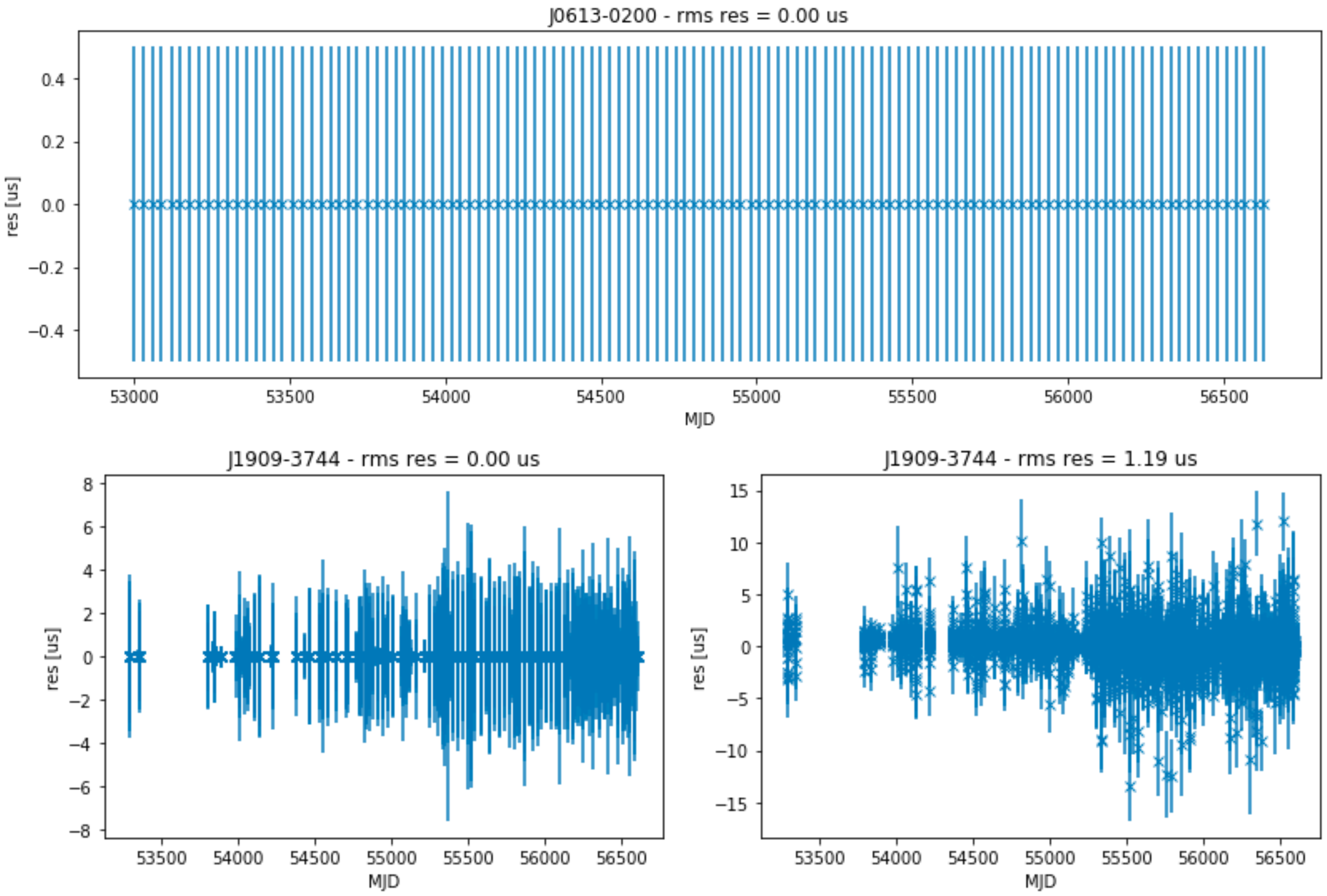
Measurement in pulsar timing



Likelihood function

$$L(\theta, \xi|\delta t) = \frac{1}{\sqrt{(2\pi)^k det(C)}} \times \exp\left(-\frac{1}{2}(\delta t - s - M\xi)^T C^{-1} (\delta t - s - M\xi) \right)$$

- C – covariance matrix, **stochastic signals**, dimensions: n x n
- бt – vector of timing residuals
- (- s – Mξ) – **deterministic signal** vector that has a known functional form for time-evolution
- Mξ – timing model signal vector
- M – design matrix, dimensions: n x m [ToA x timing model parameters]
- ξ – timing model parameters
- s – other deterministic signals, i.e. from perturbation in Solar System Ephemeris parameters

Rutger Van Haasteren, Yuri Levin, Patrick McDonald, Tingting Lu, On measuring the gravitational-wave background using Pulsar Timing Arrays, *Monthly Notices of the Royal Astronomical Society*, Volume 395, Issue 2, 11 May 2009, Pages 1005–1014, https://doi.org/10.1111/j.1365-2966.2009.14590.x

Likelihood marginalization over timing model parameters

In order not to sample timing model parameters ξ, we can assume assume a uniform prior and marginalize our likelihood over these parameters:

$$L(\theta|\delta t) = \frac{\sqrt{det(M^T C^{-1} M)^{-1}}}{\sqrt{(2\pi)^{n-m} det(C)}} \times \exp\left(-\frac{1}{2}(\delta t - s)^T C' (\delta t - s) \right)$$

$$C' = C^{-1} - C^{-1} M (M^T C^{-1} M)^{-1} M^T C^{-1}$$

Rutger Van Haasteren, Yuri Levin, Patrick McDonald, Tingting Lu, On measuring the gravitational-wave background using Pulsar Timing Arrays, *Monthly Notices of the Royal Astronomical Society*, Volume 395, Issue 2, 11 May 2009, Pages 1005–1014, https://doi.org/10.1111/j.1365-2966.2009.14590.x

Correlations in pulsar timing

- No correlations, white noise
- Between measurements
  - Red noise
  - DM variations
- Between pulsars
  - Clock error (monopole)
  - Error in Solar System Ephemeris (dipole)
  - Stochastic gravitational-wave background (quadrupole)

Dominant computational cost: C^{-1}

Likelihood

- We can further reduce computational cost by rewriting our likelihood:

$$L(\theta|\delta t) = \frac{1}{\sqrt{(2\pi)^{n-m} det(G^T C G)}} \times \exp\left(-\frac{1}{2}(\delta t - s)^T G (G^T C G)^{-1} G^T (\delta t - s) \right)$$

Where G obtained through the singular value decomposition of design matrix M:

M = U S V*

S – singular values of M

U, V – unitary matrices

Rutger van Haasteren, Yuri Levin, Understanding and analysing time-correlated stochastic signals in pulsar timing, *Monthly Notices of the Royal Astronomical Society*, Volume 428, Issue 2, 11 January 2013, Pages 1147–1159, https://doi.org/10.1093/mnras/sts097

Covariance matrix – stochastic signals

- White noise: diagonal covariance matrix, the inverse of elements:

$$\sigma_s^2 = \mathtt{T2EFAC}^2(\sigma^2 + \mathtt{T2EQUAD}^2)$$

$$\sigma_s^2 = (\mathtt{EFAC}\,\sigma)^2 + \mathtt{EQUAD}^2$$

- Red noise: off-diagonal terms are non zero
  - We model PSD of a stochastic process, P(f), and construct a covariance matrix, given parameters of that PSD model. For example:

$$\mathcal{C}(\tau) = \int_0^\infty P(f) \cos \tau f \, \mathrm{d} f$$ – Weiner-Khinchin theorem

  - τ – time delay between measurements of timing residuals

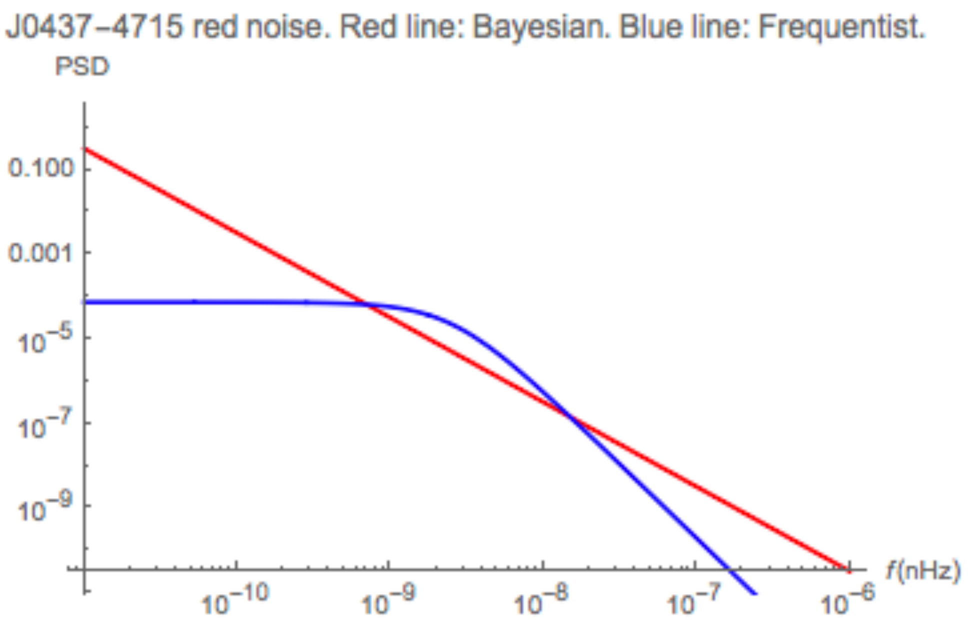
Red noise modelling

$$P(f) = \frac{P_0}{\left[1 + \left(\frac{f}{f_c}\right)^2\right]^{\alpha/2}},$$

Lorentzian power-law

$$P(f) = \frac{A^2}{12\pi^2}\mathrm{yr}^3 \left(\frac{f}{\mathrm{yr}^{-1}}\right)^{-\gamma}$$

Power-law



In our analysis represented in covariance matrix C, as a component:

K = F Φ F^T

Where Φ – defines a power-law

F – Fourier transformation matrix (Fourier design matrix)

Then we can do a faster inversion of C:

$$(N + F\Phi F^T)^{-1} = N^{-1} - N^{-1}F(\Phi^{-1} + F^T N^{-1} F)^{-1} F^T N^{-1}$$

Red noise process example (DM noise)