Tracing a TOA back from an observatory to the emission time at the pulsar involves a chain of corrections

\[ t_{a}^{psr} = t_{a}^{obs} - \Delta_{\odot} - \Delta_{IS} - \Delta_{B} \]
all TOAs are referenced to the **quasi-inertial frame of the SSB** (need Roemer delay)

Roemer delay dependent on **masses & orbits** of all important dynamical objects

don’t need SSB to navigate probes to planets (accurate SSB is not a big priority)

the **Roemer is not fit for in Tempo2**, it is subtracted from pre-fit JPL solutions
The Solar-System Ephemeris

Roemer delay

\[ \Delta_\odot = -\frac{\vec{r}_{\text{obs}} \cdot \vec{R}_{BB}}{c} \]

Observatory position

\[ \vec{r}_{\text{obs}} = \vec{r}_{\text{SSB-EB}} + \vec{r}_{\text{EB-obs}} \]

Barycenter position dependent on **masses** & **orbits** of all important dynamical objects

Small error in barycenter position

\[ \delta \Delta_\odot = \frac{\vec{e}(t) \cdot \vec{R}_{BB}}{c} \]
<table>
<thead>
<tr>
<th>Ephemerides</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE421</td>
<td>includes updates to Saturn’s orbit. Dominant uncertainty likely to be Jupiter</td>
</tr>
<tr>
<td>DE430</td>
<td>includes updates to Mercury’s orbit. Dominant uncertainty still likely to be Jupiter</td>
</tr>
<tr>
<td>DE435</td>
<td>created in Jan 2016 for Cassini, this is an incremental improvement to Saturn</td>
</tr>
<tr>
<td>DE436</td>
<td>incremental improvement to DE435</td>
</tr>
<tr>
<td>DE438</td>
<td>includes Juno corrections for Jupiter</td>
</tr>
</tbody>
</table>
JPL Ephemerides

J1713+0747 (quadratic subtracted)

Credit: M. Vallisneri

Roemer difference [ns]

Δ MJD

DE430 – DE421
DE435 – DE430
DE436 – DE435

Credit: M. Vallisneri
J1713+0747 (quadratic subtracted)

GWB amplitude of \( \sim 1e-15 \) translates to a post-fit RMS of \( \sim 75 \) ns in an 11yr dataset — van Haasteren & Levin (2013)
NANOGrav 11-year Results

**upper limits** and **detection statistics** are sensitive to our choice of ephemeris model

...mitigated by our **new Bayesian ephemeris uncertainty model**

---

**Observed GW Frequency, $f$ [Hz]**

- **blue** = common red process
- **orange** = H&D red process

---

**Characteristic Strain, $h_c(f)$**

- Pessimistic [e.g. Sesana et al. (2016)]
- Moderate [e.g. Simon & Burke-Spolaor (2016)]
- Optimistic [e.g. McWilliams et al. (2014)]

---

**Solar-system Ephemeris**

- DE421
- DE430
- DE435
- DE436
Modeling Ephemeris Uncertainties

Current Bayesian Model

Expanded Model

GOAL

marginalize over ephemeris differences to isolate GW signal from choice of DE—
Modeling Ephemeris Uncertainties

ephemeris uncertainty term

- physically motivated
  - Fourier expansion of barycenter error vector [Lentati, Taylor, Mingarelli et al. (2015)]
  - planet mass perturbation [Champion et al. (2010)]
  - dipolar spatially-correlated red process [Tiburzi et al. (2016)]

- phenomenological
  - Roemer mixture model
  - PCA of Roemer delays from DE421, DE430, etc. to construct empirical basis
  - [maybe] PCA of Roemer delays from many, many perturbed ephemerides
Modeling Ephemeris Uncertainties

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Modeling Ephemeris Uncertainties

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  - [Tiburzi et al. (2016)]

- Roemer mixture model
- PCA of Roemer delays from DE421, DE430, etc. to construct empirical basis
- PCA of Roemer delays from many, many perturbed ephemerides

“I have not failed. I've just found 10,000 ways that won't work.”

Thomas A. Edison
Physical Ephemeris Uncertainty Model

Model is 11-D

1. frame drift-rate about ecliptic “z”
1. Jupiter mass perturbation \((\text{constrained by IAU prior})\)
1. Saturn mass perturbation \((\text{constrained by IAU prior})\)
1. Uranus mass perturbation \((\text{constrained by IAU prior})\)
1. Neptune mass perturbation \((\text{constrained by IAU prior})\)
6. Jupiter orbital element perturbations

\begin{align*}
(1) & \quad \text{semi-major axis} \\
(2) & \quad \text{eccentricity} \\
(3) & \quad \text{inclination} \\
(4) & \quad \text{longitude of the ascending node} \\
(5) & \quad \text{longitude of perihelion} \\
(6) & \quad \text{mean longitude}
\end{align*}
Physical Ephemeris Uncertainty Model

- 36 pulsars
- 11 years
- equally sampled w/ 500 ns precision
- dataset created under DE436

---

moderate GWB injection

---

- dashed = no ephemeris uncertainty modeling
- solid = physical ephemeris uncertainty model

---

![Graph showing distributions of log_{10} A_{GW} with different datasets and models.](image-url)
Physical Ephemeris Uncertainty Model

11-year dataset simulations
(created with exactly the same pulsars, noise properties, and sensitivity as the real dataset)

Uninformative Jupiter orbit priors
Physical Ephemeris Uncertainty Model

11-year Uninformative Jupiter orbit priors

Upper limit (for all connected) $\sim 1.34 \times 10^{-15}$

[Graph showing log scale for $A_{GWB}$]
Physical Ephemeris Uncertainty Model

11-year

**colors**
- purple = prior distribution
- blue = DE435 (uninformative prior)
- orange = DE436 (uninformative prior)
- green = DE435 (JPL prior)
- red = DE436 (JPL prior)
Physical Ephemeris Uncertainty Model

11-year

color explanations:
purple = prior distribution
blue = DE435 (uninformative prior)
orange = DE436 (uninformative prior)
green = DE435 (JPL prior)
red = DE436 (JPL prior)

Set III celestial mechanics coordinates

- $\Delta l_0 + \Delta r \times 10^{-6}$
- $\Delta p \times 10^{-6}$
- $\Delta q \times 10^{-7}$
- $e \Delta r \times 10^{-7}$
- $\Delta a/a \times 10^{-7}$
- $\Delta e \times 10^{-7}$
Physical Ephemeris Uncertainty Model

Uninformative Jupiter orbit priors

9-year

Upper limit (for all connected) ~ 2.7e-15
BayesEphem does not hinder detection prospects

Injection SSE: DE436
Recovery SSE: DE430

$β$ [3A vs. 2A] vs. $A_{\text{GWB}}$ [injected]
BayesEphem is easy to use

accessible through “enterprise”
github.com/nanograv/enterprise

```python
# red noise
s = red_noise_block(prior=amp_prior, Tspan=Tspan, components=components)

# common red noise block
s += common_red_noise_block(psd=psd, prior=amp_prior, Tspan=Tspan,
                                 components=components, gamma_val=gamma_common,
                                 name='gw')

# ephemeris model
if bayesephem:
    s += deterministic_signals.PhysicalEphemerisSignal(use_epoch_toas=True)

# timing model
s += gp_signals.TimingModel()

# adding white-noise, and acting on psr objects
models = []
for p in psrs:
    if 'NANOGrav' in p.flags['pta'] and not wideband:
        s2 = s + white_noise_block(vary=False, inc_ecorr=True, select=select)
        models.append(s2(p))
    else:
        s3 = s + white_noise_block(vary=False, inc_ecorr=False, select=select)
        models.append(s3(p))

# set up PTA
pta = signal_base.PTA(models)
```